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Advances in Nonlinear Dynamics and Vibrations of the Body with Constant and Variable Mass

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Abstract. In this paper the overview of investigation of the author in the field of nonlinear dynamics and vibrations of the body with constant and variable mass is given. The main contributions of the author in the field of systems with variable mass are presented. The both: continual and discontinous mass variation is analysed. The reactive force and the newly introduced reactive torque, which are evident for continual time variable mass, are discussed. Besides, the dynamics of the rotating machine due to finite mass variation is treated. The mass change is the consequence of the break in rotor. The method for obtaining the dynamic properties of the remaining and of the separated body is developed. Vibration, as the side effect of the main motion, is also analysed. The obtained vibration results are applicable for extending the knowledge in diagnostics of machine fail.

Keywords: reactive force; reactive torque; break in rotor; vibrodiagnostics, .

1 Introduction

In 1687, Ser Isac Newton (1642-1727) published three laws [1] which are the basic for the classical mechanics. The laws are written in Latin language in the form of sentences without mathematical formulation. These are:
Lex I: Corpus omne perseverare in statu suo quiescendi vel movendi uniformiter in directum, nisi quatenus illud a viribus impressis cogitur statum suum mutare.
Lex II: Mutationem motus proportionalem esse vi motrici impressae, et fieri secundum lineam rectam qua vis illa imprimitur.
Lex III: Actioni contrariam semper et eaqualem esse reactionem: sive corporum duorum actiones in se mutuo semper esse eaquales et in partes contrarias dirigi.
Here, the comment on the second law, which obtains a lot of mathematical treatments, will be given: It is usually assumed that the 'mutationem motus' is given with acceleration $\vec{a}$ which direction corresponds to the force $\vec{F}$. The connection between force and acceleration is given with a parameter *m* which Gustav Kirchoff (1824-1887)

called 'mass'. Then, the expression of the second Newton's law for the particle with constant mass is $\vec{F} = m\vec{a}$. However, even before Newton it was known that the mass need not be constant. Thus, Galileo Galilei (1564-1642) saw some anomalies in motion of the Moon, for which Pierre-Simon Laplace (1749-1827) said that are due to secular acceleration. The German scientists A. Oppalzer [2] concluded that this phenomena is caused by increasing of the mass of the Earth and Moon. For a 100 year a layer with thickness of 2.8 mm is formed on the Earth's surface. It was the proof of the previously published result of the French scientist Ch. Dufour [3] that the dust which comes to the Earth from the space increases its mass. For one year on the surface of France falls about 0.1 m^3 dust. I. Meshcherski (1859-1935) was the first to extend the Newton's law for the case when mass parameter is not constant.
In this paper the results of the author of the paper based on the equation of Meshcherski for mass variable systems are shown.

2 Dynamics of Bodies with Continual Mass Variation

Meshcherski investigated the dynamics of a particle whose mass is continually varying in time. He concluded that on the particle with variable mass beside the well known forces an additional force acts, which he called 'reactive force'. It is the product of the mass variation in time and the relative velocity of separating mass [4]. The knowledge about this force was of fundamental interest for developing of the interplanet rocket technique. The Rssian scientist Konstantin Ciolkovski (1857-1935) applied the result of Meshcherski for obtaining of the maximal direct velocity of the rocket due to fuel mass variation and reactive force. It depends on the initial velocity of the rocket, velocity of fuel combustion and the relation between the initial and final mass of the rocket, but is independent on the mass variation law. The formula of Ciolkovski gives us the possibility to design the rocket with velocity higher than the first cosmic velocity (7.9 km/s) if the rocket has to have an elliptic trajectory around the Earth, or higher than the second cosmic velocity (11.2 km/s) when the rocket gets out of the Earth's gravitational field or higher than the third cosmic velocity (16.7 km/s) if the rocket gets out of the Sun's galactic system.

Based on the consideration given by Meshcherski [5], we developed the dynamics of bodies with variable mass and variable moment of inertia. It is concluded that beside the reactive force a reactive torque occurs due to continual variation of the mass and of the moment of inertia of the body. The reactive torque is the product of the variation of the moment of inertia in time and of the relative angular velocity of the separated body [6]. Usually, in analysis of real machines with variable mass and moment of inertia the reactive force and the reactive torque are neglected as it is assumed that the influence of these functions is quite small due to slow mass variation. To disallow this statement we investigated the influence of the reactive force on the motion of a one degree-offreedom oscillator. In **Fig. 1** three oscillators with equal mass variation but different velocity of mass separation are shown. It is concluded that the reactive force has a significant influence on the motion. For various velocities of mass variation the amplitude of vibration may increase (**Fig. 1.a**), decrease (**Fig. 1.b**) or remain constant (**Fig. 1.c**).

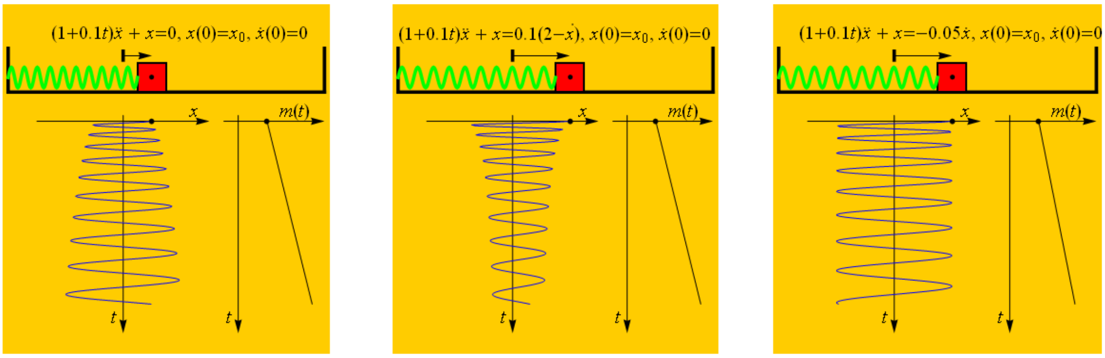

Fig. 1. Motion of the oscillator with variable mass: a) without reactive force, b) with reactive force and c) with special value of the reactive force.

Based on the general dynamics of bodies with variable mass the dynamics of rotors is developed [7]. There are a lot of machines in cable, carpet, paper, textile industry etc. where the mass, geometry and moment of inertia are continually changed in time due to winding up of the band (**Fig. 2.a**). The model of the rotor is a shaft-disc system where the mass of the time is varying in time (**Fig. 2.b**.)

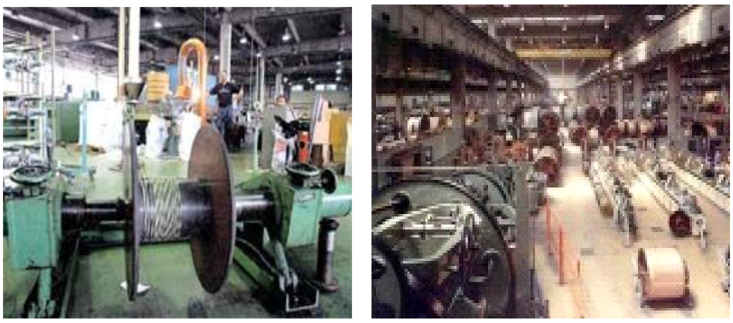

Fig. 2.a) Rotors with continual mass variation.

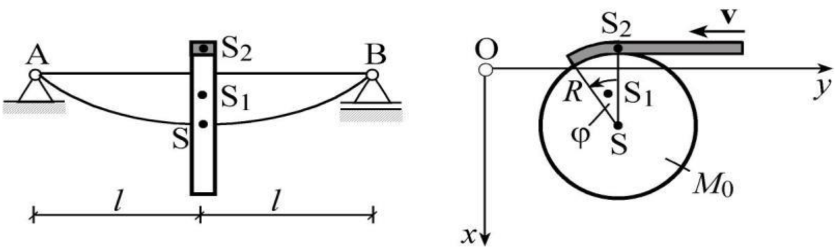

Fig. 2.b) Model of the rotor with variable mass.

Mathematical model is a system of two coupled differential equations with time variable parameters. In our consideration the deflection function is introduced in the complex form which gives us the opportunity to give solution for the special kind of nonlinearity in shaft.

3 Dynamics of Bodies with Discontinual Mass Variation

Mass of the body is changed if a part is separated during the motion. This failure is specific for the rotors where the separation is caused by fatigue or defect in material and sometimes due to impact (**Fig. 3**).

Fig. 3. Failure in a fan.

It is the question how will operate after crash the remainder part of the rotor and how will behave the separater part. Unfortunately, investigation in this problem are rather few. In the paper [8] the theoretical consideration of the motion of the discontinual mass variation is given. Based on these results it is concluded that the dynamics of body separation is oposite to the plastic impact between two bodies [9]. It is proved that the Carnot theorem about energy distribution in impact is valied in the plastic separation of the body into two parts, too. This result is important to be known for further investigation in the matter.

An example of body separation in the wind turbine is worth to be mentioned (**Fig. 4**). Using the developed theoretical consideration it is obtained that the angular velocity of the rotor is increased up to 50% if one wing is separated.

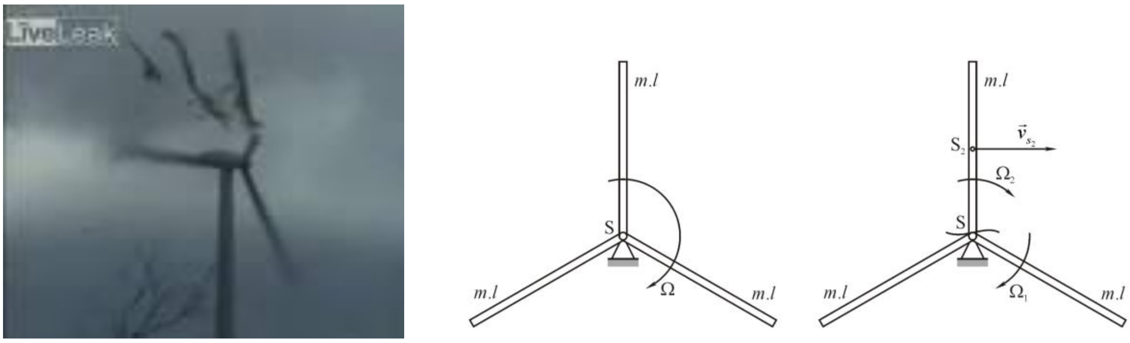

Fig. 4. a) Separation of a wing of a wind turbine, b) model of wing separation.

The separated wing has the plane motion in the air. The reach of the wing on the earth depends on the initial velocity, its position at the moment of crash and geometric and mass properties. It is worth to say that the position of fall of the wing can be approximately determined. It is a very important date for safety reasons.

4 Vibrations and Vibrodiagnostics

In the rotating systems vibration occur. The part of the useful energy of rotation is disspated on the vibrations. To reduce the energy separation it is necessary to analyze the vibration of the system. As the properties of the material of the rotor and the geometry of the system is nonlinear, the vibrations are also nonlinear. Mathematical model of vibrations are strong nonlinear differential equations. The aim is to find the solution for these equations. We developed a significant number of analytic asymptotic methods for solving strong nonlinear differential equations [10], where most of them are based on the exact analytic solution of the pure nonlinear second order differential equations. The solution has the form of the Ateb function which is represents the complete inverse beta function. Two of the methods we developed are widely applied: one, which is called 'Cveticanin method' [11] and is suitable for equations with damping and the second is the homotopy perturbation method for the strong nonlinear differential equations [12] (which is highly citated). The advantage of both methods is that they give the analytical solutions which are accurate in a wide time interval independently on the value of nonlinear and initial conditions (see **Fig. 5**).

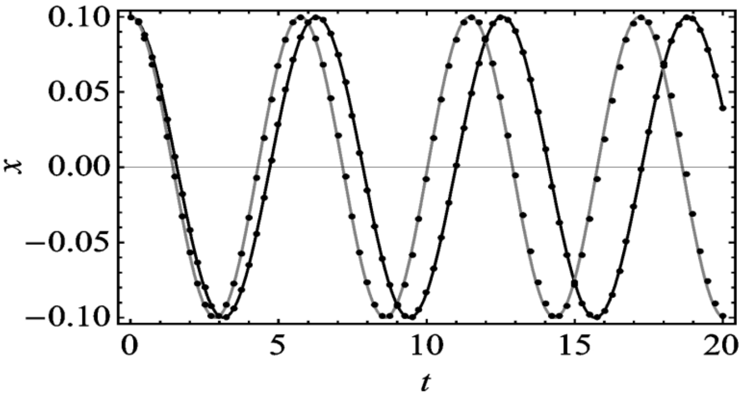

Fig.5. Comparison of analytical (full line) and numerical (dotted line) solutions for two types of nonlinear oscillators: pure cubic (black line) and pure quadratic (grey line).

The knowledge in nonlinear vibrations is valuable in diagnostics of failures in rotating machines. Based on the measured velocity of vibration and frequency analysis it is possible to signify the problem in machine operation without stopping it and destroying the system [13]. Thus, beside the unbalance in rotors and misalignment between rotor and shaft, we can predict failor on the tooth of the gear-wheel (**Fig. 6.a**), accidence in the bearing ring (**Fig. 6.b**) or balls, even initial cracks in the shaft.

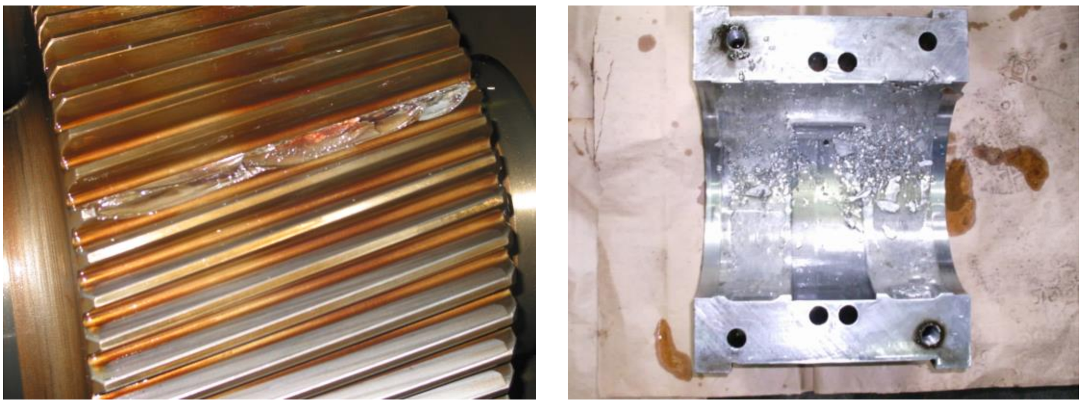

Fig. 6. Failer on: a) tooth of the gear-wheel, b) bearing ring.

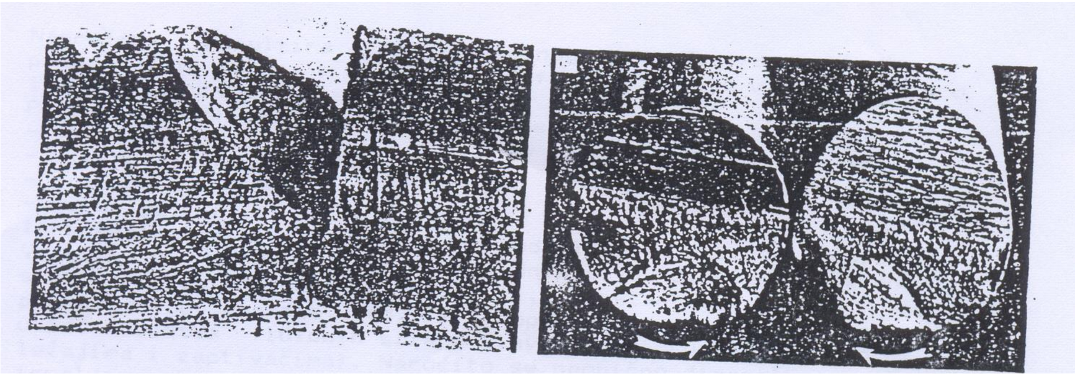

Fig.7. Crack in the turbine shaft.

The correctness of diagnostics and prediction of rotor behavior depends on our knowledge in vibration analysis.

4 Direction for Future Investigation

Instead of Conclusions the direction for future investigation is given. Based on the results of our investigation we suggest two new research directions: one, to develope energy harvester based on ambiental vibration, and the second, to compose new materials and structures for vibration and acoustic elimination.

1. Recently, microelectromechanical systems (MEMS), which represent the couplings between electric systems and mechanicol oscillators, have a very wide range of application due to their function and dimensions. They are incorporated in sensors, actuators, accelerators, flying objects [14], etc. These systems need quite small amount of energy, but the energy supply has to be continual. Usually, the battery is used, but it has many disadvantages due to the working life and chemical reaction in them. It would be very effective to have energy harvester, based for example on piezoelectric element, which would transform the ambiental vibration into electric energy.

2. Nowadays, one of the most serious problem is the pollution of the environment with vibration and noise (which is the additional effect of vibration). Because of that the vibration and noise have to be decreased or even eliminated. Nowadays, using the knowledge in vibrations a new method for elimination of the noise with certain frequency is suggested. The idea is to create the new so called acoustic metamaterial which will be suitable to fullfill such a requirement. The basic element of the metamaterial sturcture is the conventional well-known vibration absorber. Connecting these elements in a specific way the metamaterial acoustic beams are formed which are able to give acoustic gap for certain frequency band.

Acknowledgement

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